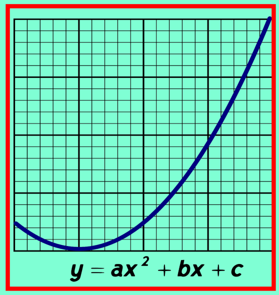
Math 125

Spring 2021

Lecture 4



Class QZ 3

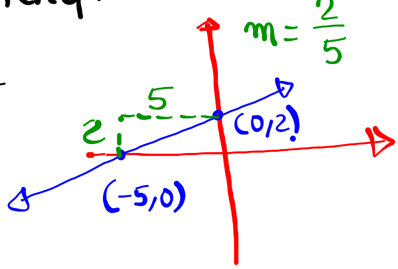
Portrait style only.

1) Graph

$2x - 5y = -10$

by intercept Method.

x	y
0	2
-5	0

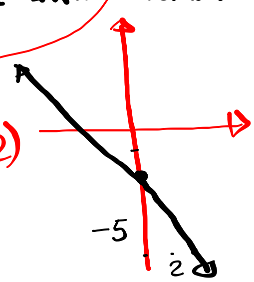


2) Graph

$y = \frac{-5}{2}x - 2$ using Slope-Int. method

$m = \frac{-5}{2}$

Y-Int (0,-2)



Write $\underbrace{4x-3y=9}_{\text{Standard Form}}$ in $\underbrace{\text{Slope-Int. Form}}_{y=mx+b}$, then graph.

Isolate Y $\Rightarrow$ $-3y=-4x+9$

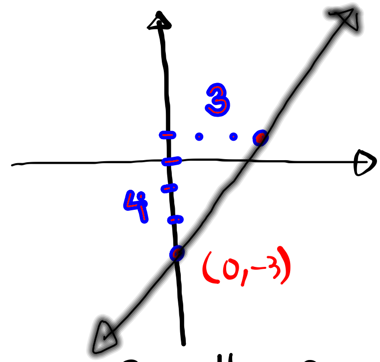
$$\frac{-3}{-3}y=\frac{-4}{-3}x+\frac{9}{-3}$$

$$\boxed{y=\frac{4}{3}x-3}$$

$$\Downarrow$$

Function Notation $\rightarrow$ $\boxed{F(x)=\frac{4}{3}x-3}$

Replace Y with f(x)



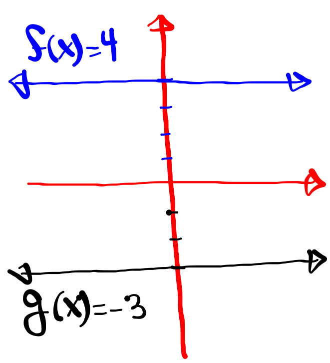
$f(x)=\frac{4}{3}x-3$

So f(x) & y are interchangeable.

Graph $\underbrace{f(x)=4}_{\text{Constant Function}}$ $\Rightarrow$ $y=4$ Horizontal line

$\Downarrow$ $\Downarrow$

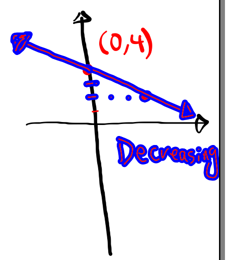
Graph $g(x)=-3$ $\Rightarrow$ $y=-3$



Graph $\underbrace{f(x) = \frac{-2}{3}x + 4}_{\text{Linear Function}} \Rightarrow \underbrace{y = \frac{-2}{3}x + 4}_{\text{Slope-Int. Form}}$

Y-Int $(0,4)$

$m = \frac{-2}{3}$



Graph $g(x) = \frac{1}{2}x + 2$

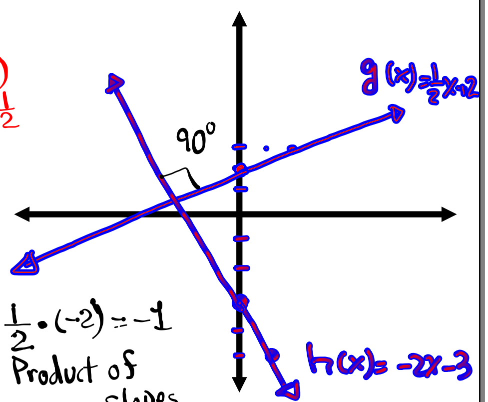
and $h(x) = -2x - 3$ in the Same Coordinate System.

$y = \frac{1}{2}x + 2$ Y-Int $(0,2)$ slope $m = \frac{1}{2}$

$y = -2x - 3$

Y-Int $(0,-3)$

$m = -2 = \frac{-2}{1}$



$\frac{1}{2} \cdot (-2) = -1$

Product of slopes

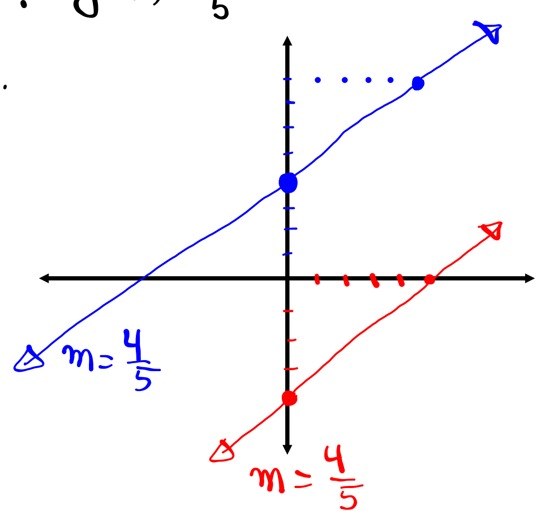
Graph $f(x) = \frac{4}{5}x - 4$ & $g(x) = \frac{4}{5}x + 4$ in the Same Coordinate System.

$f(x) = \frac{4}{5}x - 4$

$m = \frac{4}{5}$, Y-Int $(0,-4)$

$g(x) = \frac{4}{5}x + 4$

$m = \frac{4}{5}$, Y-Int $(0,4)$



Same slope
Different Y-Int $\Rightarrow$ Parallel lines.

Consider $f(x) = x^3 + 8$

Find

1) $f(0)$
$= 0^3 + 8$
$= 0 + 8 = \boxed{8}$ $(0,8)$

2) $f(2)$
$= 2^3 + 8$
$= 8 + 8$
$= \boxed{16}$ $(2,16)$

3) $f(-2)$
$= (-2)^3 + 8$
$= -8 + 8$
$= \boxed{0}$ $(-2,0)$

4) $f(3x^2)$
$= (3x^2)^3 + 8$
$= 3^3 \cdot (x^2)^3 + 8$
$= 27\, x^{2 \cdot 3} + 8$
$= \boxed{27x^6 + 8}$ ✓
$(3x^2, 27x^6 + 8)$

Exponential Rules
$(xy)^n = x^n y^n$
$(x^m)^n = x^{m \cdot n}$

Consider $f(x) = x^2 - 4x + 4$

Find

1) $f(0) = 0^2 - 4(0) + 4$
$= \boxed{4}$ $(0,4)$

2) $f(2) = 2^2 - 4(2) + 4$
$= 4 - 8 + 4$
$= \boxed{0}$ $(2,0)$

3) $f(-3) = (-3)^2 - 4(-3) + 4$
$= 9 + 12 + 4$
$= \boxed{25}$ $(-3,25)$

4) $f(x+2)$
$= (x+2)^2 - 4(x+2) + 4$
$= \underbrace{(x+2)(x+2)}_{\text{foil}} \underbrace{-4(x+2)}_{\text{dist.}} + 4$
$= x^2 + \cancel{2x} + \cancel{2x} + \cancel{4} - \cancel{4x} - \cancel{8} + \cancel{4}$
$= \boxed{x^2}$ $(x+2, x^2)$

$f(x) = |2x - 6| + 4$

1) $f(0) = |2(0) - 6| + 4$
$= |0 - 6| + 4$
$= |-6| + 4 = 6 + 4 = \boxed{10}$

2) $f(3)$
$= |2 \cdot 3 - 6| + 4$
$= |6 - 6| + 4 = |0| + 4 = 0 + 4$
$= \boxed{4}$

3) $f(-3)$
$= |2(-3) - 6| + 4$
$= |-6 - 6| + 4$
$= |-12| + 4 = 12 + 4$
$= \boxed{16}$

3) $f(\frac{1}{2}x + 3)$
$= |2(\frac{1}{2}x + 3) - 6| + 4$
$= |2 \cdot \frac{1}{2}x + 2 \cdot 3 - 6| + 4$
$= |x \cancel{+6} \cancel{-6}| + 4$
$= \boxed{|x| + 4}$

$f(x) = \dfrac{x - 6}{x + 3}$ Find

1) $f(0) = \dfrac{0 - 6}{0 + 3} = \dfrac{-6}{3}$
$= \boxed{-2}$

2) $f(3) = \dfrac{3 - 6}{3 + 3} = \dfrac{-3}{6} = \boxed{\dfrac{-1}{2}}$

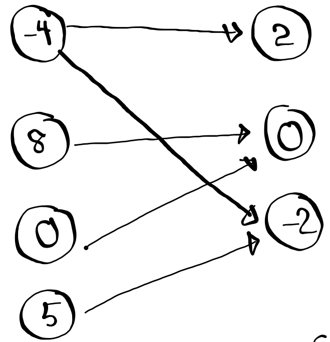
3) $f(6) = \dfrac{6 - 6}{6 + 3} = \dfrac{0}{9}$
$= \boxed{0}$

$\dfrac{\text{Zero}}{\text{NonZero}} = \text{Zero}$

4) $f(-3) = \dfrac{-3 - 6}{-3 + 3}$
$= \dfrac{-9}{0}$

$\dfrac{\text{Nonzero}}{\text{Zero}} \rightarrow \boxed{\text{Undefined}}$

Consider the display below



where arrow begins
x-vaues

and where it points to
Y-values.

(-4,2), (-4,-2),
(8,0), (0,0),
(5,-2)

x-values ⇒ domain = {-4, 8, 0, 5}

Y-values ⇒ Range = {2, 0, -2}

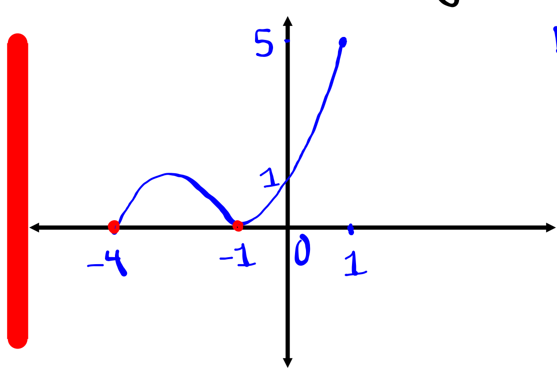
Do You think we have a function here? Explain.

NO

(-4,2) & (-4,-2)

No input values can have more than one output value.

Consider the drawing below:



Domain ⇒ $-4 \le x \le 1$ ⇒ $[-4, 1]$

Range ⇒ $0 \le y \le 5$ ⇒ $[0, 5]$

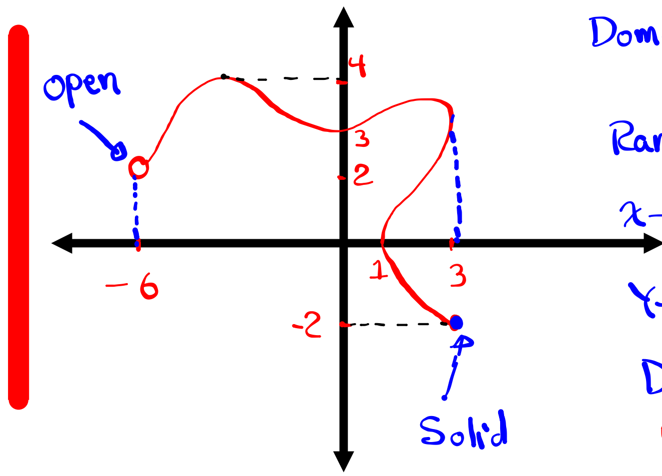
x-Ints ⇒ (-4,0) & (-1,0)

Y-Int ⇒ (0,1)

Does this graph belong to a function? Explain.

Try Vertical Line Test ⇒ It passes V.L.T. ⇒ Yes, It belongs to a function.

Consider the graph below



Domain $(-6, 3]$ ← Interval notation

Range $[-2, 4]$

x-Ints $(1, 0)$ ← Point

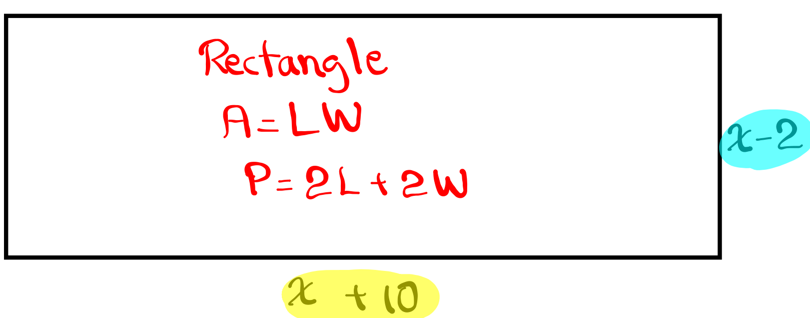
Y-Ints $(0, 3)$

Discuss Function or not.

NO, It Fails the V.L.T.

Does not belong to a Function

Find area & Perimeter of



$A = LW = (x+10)(x-2) = x^2 - 2x + 10x - 20$

Foil

$= \boxed{x^2 + 8x - 20}$

$P = 2L + 2W = 2(x+10) + 2(x-2) = 2x + 20 + 2x - 4$

$= \boxed{4x + 16}$

Find Area & Perimeter

Square
$A = S^2$
$P = 4S$

$2x - 3$

$A = S^2 = (2x - 3)^2$

$= (2x - 3)(2x - 3)$

$= 4x^2 - 6x - 6x + 9$

$= \boxed{4x^2 - 12x + 9}$

$P = 4S = 4(2x - 3) = \boxed{8x - 12}$

Factor Completely

1) $3x + 12$

$= \boxed{3}x + \boxed{3} \cdot 4$

$= \boxed{3(x + 4)}$ ✓

2) $2x^2 - 8x$

$= 2 \cdot x \cdot x - 2 \cdot 2 \cdot 2 \cdot x$

$= \boxed{2x(x - 4)}$

3) $x^2 + 7x + 12$

$\boxed{(x + 3)(x + 4)}$

1, 12
2, 6
3, 4

4) $x^2 + x - 12$

$\boxed{(x + 4)(x - 3)}$

Zero-Product Rule
Zero-Factor Property

$\Rightarrow$ If $A \cdot B = 0$, then $A = 0$ or $B = 0$ (May be both)

Solve $(x-2)(x+6)=0$

by Zero-Product Rule

$x-2=0$ OR $x+6=0$

$\boxed{x=2}$ OR $\boxed{x=-6}$

Solution Set $\{-6, 2\}$

Solve $(2x+3)(2x-3)=0$ by Zero-Factor theorem.

by Z.F.T.

$2x+3=0$ OR $2x-3=0$

$2x=-3$ $\quad$ $2x=3$

$\boxed{x=\frac{-3}{2}}$ $\quad$ $\boxed{x=\frac{3}{2}}$

$\{\pm\frac{3}{2}\}$

Solve $x^2 - 24 = 2x$

Hint: Make RHS Zero.

$x^2 - 24 - 2x = 0$

write in descending order

$x^2 - 2x - 24 = 0$

1, 24
2, 12
3, 8
4, 6

Factor the LHS Completely

$(x + 4)(x - 6) = 0$

By Z.P.R

$x + 4 = 0$ OR $x - 6 = 0$

$\boxed{x = -4}$ $\boxed{x = 6}$

$\{-4, 6\}$

looking ahead

$f(x) = x + 5$ $g(x) = x - 5$

Find

$f(x) + g(x) = x \cancel{+5} + x \cancel{-5} = \boxed{2x}$

$f(x) - g(x) = x + 5 - (x - 5)$

$= \cancel{x} + 5 \cancel{-x} + 5 = \boxed{10}$

$f(x) \cdot g(x) = (x + 5) \cdot (x - 5) =$

FOIL

$= x^2 \cancel{-5x} \cancel{+5x} - 25$

$= \boxed{x^2 - 25}$

Be aware of SG 2 due dates.

Work on SG 3 as well.

Class QZ 4

Box Your Final Ans.

$f(x) = x^2 - 4x$ Find

1) $f(0)$

2) $f(4)$

3) $f(-4)$

4) $f(x+2)$